



## FINAL TEST SERIES JEE -2020

## TEST-03 ANSWER KEY

Test Date :22-12-2019

## [PHYSICS]

1. 
$$n' = \left( \frac{v + v_0}{v - v_s} \right) \times n$$
- $$= \left( \frac{v + 0.2v}{v - 0} \right) \times n = \frac{1.2v}{v} \times n$$
- $$= 1.2n = 1.2f$$
- since, the source is stationary, therefore apparent wavelength remains unchanged, i.e.,  $\lambda$ .

2. 255 : 425 : 595  
51 : 85 : 117  
3 : 5 : 7  $\therefore$  COP  
where  $3n_c = 255$  Hz

3. 
$$\frac{\Delta n}{n} = \frac{1}{2} \frac{\Delta T}{T}$$

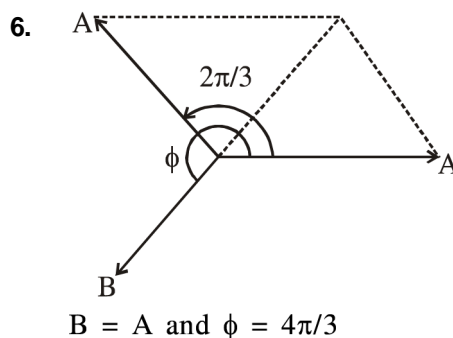
$$\frac{5}{n} \times 100 = \frac{1}{2} \times 2\%$$

$$n = 500 \text{ Hz}$$

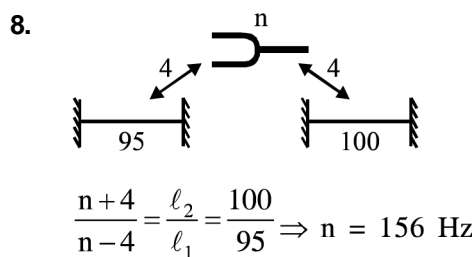
4. 
$$\frac{n_1}{n_2} = \frac{\ell_2}{\ell_1} \Rightarrow \frac{800}{1000} = \frac{\ell_2}{50} \Rightarrow \ell_2 = 40 \text{ cm}$$

5. 
$$\frac{T}{2} = 0.5 \Rightarrow T = 1 \text{ s} \Rightarrow n = 1$$

$$\text{Wavelength } \lambda = \frac{v}{n} = 10 \text{ m}$$



7. B



9. Frequency =  $\frac{1}{\text{time}}$  [Velocity =  $\frac{\text{mean free path}}{\text{time}}$ ]

$$f = \frac{v_{\text{rms}}}{\lambda_m} = \frac{v_{\text{rms}}}{2\ell} = \frac{200}{2+5} = 20 \text{ s}^{-1}$$

10. D

11. KE of 1 g gas =  $\frac{f RT}{2 M_w}$

For diatomic gas  $f = 5$ , and for  $\text{O}_2$   $M_w = 32 \text{ g}$ .

$$\text{So KE of 8 g gas} = 8 \times \frac{5}{2} \times \frac{RT}{32} = \frac{5}{8} RT$$

$$12. \frac{\Delta\phi}{360^\circ} = \frac{\Delta\lambda}{\lambda}$$

$$\text{here } \Delta\lambda = 2\text{cm}$$

$$K = \frac{2\pi}{10}\text{cm} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 10\text{cm}$$

$$\text{So } \frac{\Delta\phi}{360^\circ} = \frac{2}{10} \Rightarrow \Delta\phi = 72^\circ$$

13. Wave speed does not depend on "freq."

14. C

15.  $\ell$  effectively increases. Due to shifting of centre of mass.

$$\therefore T \propto \sqrt{\ell}$$

16. A

$$17. K = \omega^2 M = \left(\frac{2\pi}{\pi/5}\right)^2 \times 10 \times 10^{-3} = 1 \text{ N/m}$$

$$F_{\max} = -KA = -1 \times 0.5 = 0.5\text{N}$$

18. D

19. B

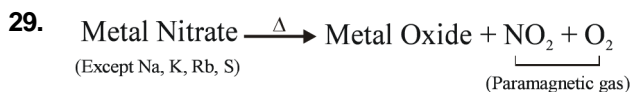
20. A

## [CHEMISTRY]

26. B

27. A

28. C



30. C

$$31. \left[ \text{Ionic mobility} \propto \frac{1}{\text{Hydration effect}} \right]$$

32. d-block element in higher oxidation state shows acidic nature.

33. D

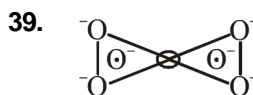
34. D

35. B

36. D

37. B

38. B



O  $\rightarrow$  Oxygen

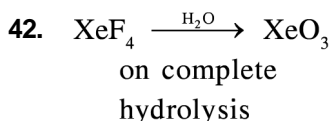
$(\text{Si}_2\text{O}_7)^{-6}$

.  $\rightarrow$  Silicon

(Pyrosilicates)

40. A

41. B



43. C

44. A

45. A

46. 1

47. 5

48. 1

49. 4

50. 1

## [MATHEMATICS]

51.

Ans. (3)

We have,  $z = 0$  for the point where the line intersects the curve

$$\therefore \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$$

$$\Rightarrow x = 5 \text{ and } y = 1$$

Putting these values in  $xy = c^2$

$$\Rightarrow 5 = c^2 \Rightarrow c = \pm\sqrt{5}$$

52. Ans. (3)

D.R's of normal to plane  $x + y + z - 1 = 0$  and  $x + ky + 3z - 1 = 0$  is  $(1, 1, 1)$  and  $(1, k, 3)$  respectively

$\Rightarrow$  D.R. of normal to a plane perpendicular to given planes

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & k & 3 \end{vmatrix} = \hat{i}(3-k) - \hat{j}(2) + \hat{k}(k-1)$$

$$\begin{aligned} \Rightarrow \frac{1+2\lambda}{3-k} &= \frac{2+\lambda}{-2} = \frac{1+3\lambda}{k-1} \\ \Rightarrow -2-4\lambda &= 6+3\lambda-2k-\lambda k \\ -4-10\lambda &= 4+2\lambda \Rightarrow 12\lambda = -8 \Rightarrow \lambda = -\frac{2}{3} \\ \Rightarrow -2+\frac{8}{3} &= 6-2-2k+2-k=3 \\ \Rightarrow \frac{2}{3}-4 &= -\frac{4}{3}k \Rightarrow -\frac{10}{3} = -\frac{4}{3}k \Rightarrow k = \frac{5}{2} \end{aligned}$$

53. **Ans. (3)**

$g(x) = x|x|^3$  has 4 repeated roots  
 $\therefore g''(x)$  is cont. and diff. at  $x = 0$

$$\therefore \text{consider } f(x) = \begin{cases} x^p \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f'(0^+) = \lim_{h \rightarrow 0} \frac{h^p \sin \frac{1}{h}}{h} = 0 \quad \text{if } p > 1$$

$\therefore f'(0) = 0$  for  $p > 1$

$$\therefore f'(x) = \begin{cases} p x^{p-1} \sin \frac{1}{x} - x^{p-2} \cos \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$f''(0^+) = \lim_{h \rightarrow 0} \frac{p h^{p-1} \sin \frac{1}{h} - h^{p-2} \cos \frac{1}{h}}{h} = 0 \quad \text{if } p > 3$$

$$\therefore f''(x) = \begin{cases} (p(p-1)x^{p-2} - x^{p-4}) \sin \frac{1}{x} + (2x^{p-3}) \cos \frac{1}{x} & \text{for } x \neq 0 \\ = 0 & \text{for } x = 0 \end{cases}$$

$\therefore f''(x)$  to be continuous  $p \in (4, \infty)$

54. **Ans. (4)**

$$\frac{2x^3 + 3x^2 + x - 3}{x^2 + x - 2} = (2x+1) + \frac{1}{x-1} + \frac{3}{x+2}$$

$$\frac{d}{dx} \left[ \frac{2x^3 + 3x^2 + x - 3}{x^2 + x - 2} \right] = 2 - \frac{1}{(x-1)^2} - \frac{3}{(x+2)^2}$$

$$\therefore A = 2, B = -1, C = -3$$

$$\therefore A + B + C = 0$$

55. **Ans. (2)**

$$PA \times PB = (PT)^2$$

where  $PT$  = length of tangent

$$(PT)^2 = (-1)^2 + 3^2 - 2(-1) + 4(3) - 8 = 16$$

$$P(A)P(B) = 16$$

$$\therefore AM \geq GM$$

$$PA + PB \geq 8$$

56. **Ans. (1)**

The normal at the extremities of focal chord meet at right angle. So orthocentre is the point of intersection of normals.

$$\text{If } P(at_1^2, 2at_1), Q(at_2^2, 2at_2) \text{ then } t_1 t_2 = -1.$$

Point of intersection of normals

$$h = a(t_1^2 + t_2^2 + t_1 t_2 + 2)$$

$$k = -at_1 t_2 (t_1 + t_2)$$

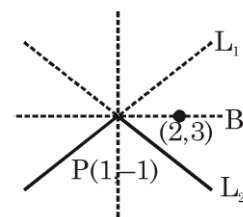
57. **Ans. (1)**

Fixed point of family is  $(1, -1)$

$\Rightarrow$  other bisector is

$$y + 1 = -\frac{1}{4}(x - 1)$$

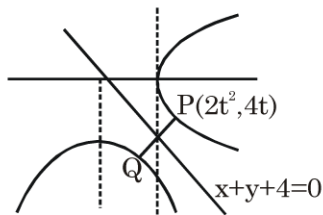
$$x + 4y + 3 = 0$$



58. Ans. (4)

Let point be  $\left(8\lambda + \frac{1}{3}, 3\lambda, -6\lambda\right)$  which also satisfies both the plane  $P_1 = 0 = P_2$

59. Ans. (1)

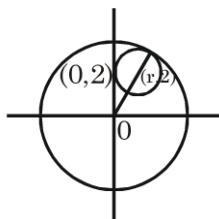


for minimum distance  $\left. \frac{dy}{dx} \right|_P = -1$   
 $\Rightarrow t = -1$   
 $\Rightarrow \text{min distance} = PQ = 2\sqrt{2}$

60. Ans. (2)

$$4 = \sqrt{r^2 + 4} + r$$

$$\Rightarrow r = \frac{3}{2}$$



61. Ans. (1)

$P_1$  and  $P_2$  are  $x + 2y - 2z = 0$   
 and  $2x - 3y + 6z = 0$

$$\cos \alpha = \left| \frac{2 - 6 - 12}{3 \cdot 7} \right| = \frac{16}{21}$$

62. Ans. (3)

Differentiate both sides wrt 'x',

$$(e - 1)e^{xy} \left( \frac{xdy}{dx} + y \right) + 2x = e^{x^2+y^2} \left( 2x + 2y \frac{dy}{dx} \right)$$

$$(e - 1) \left( \frac{dy}{dx} \right) \Big|_{(1,0)} + 2 = e(2)$$

$$\left. \frac{dy}{dx} \right|_{(1,0)} = 2$$

63. Ans. (3)

$$h(x) = f(g(f(x)))$$

$$h'(x) = f'(g(f(x))) \cdot g'(f(x)) \cdot f'(x)$$

$$h'(2) = 64.$$

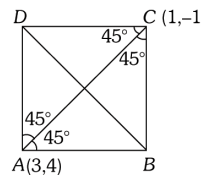
64. B

65. D Point lies on director circle of ellipse  $\frac{\pi}{2}$

66. (b) Let  $y = x^x \Rightarrow \log y = x \log x$   
 $\therefore \lim_{y \rightarrow 0} \log y = \lim_{x \rightarrow 0} x \log x = 0 = \log 1 \Rightarrow \lim_{x \rightarrow 0} x^x = 1$

67. (c) Obviously, slope of AC = 5/2.  
 Let m be the slope of a line inclined at an angle of

$$45^\circ \text{ to AC, then } \tan 45^\circ = \pm \frac{m - \frac{5}{2}}{1 + m \cdot \frac{5}{2}} \Rightarrow m = -\frac{7}{3}, \frac{3}{7}.$$



Thus, let the slope of  $AB$  or  $DC$  be  $3/7$  and that of  $AD$  or  $BC$  be  $-\frac{7}{3}$ . Then equation of  $AB$  is  $3x - 7y + 19 = 0$ .

Also the equation of  $BC$  is  $7x + 3y - 4 = 0$ .

On solving these equations, we get,  $B\left(-\frac{1}{2}, \frac{5}{2}\right)$ .

Now let the coordinates of the vertex  $D$  be  $(h, k)$ . Since the middle points of  $AC$  and  $BD$  are same, therefore

$$\frac{1}{2}\left(h - \frac{1}{2}\right) = \frac{1}{2}(3 + 1) \Rightarrow h = \frac{9}{2}, \quad \frac{1}{2}\left(k + \frac{5}{2}\right) = \frac{1}{2}(4 - 1)$$

$$\Rightarrow k = \frac{1}{2}. \text{ Hence, } D = \left(\frac{9}{2}, \frac{1}{2}\right).$$

68. (d) Let the centre be  $(h, k)$ , then radius =  $h$

$$\text{Also } CC_1 = R_1 + R_2$$

$$\text{or } \sqrt{(h-3)^2 + (k-3)^2} = h + \sqrt{9+9-14}$$

$$\Rightarrow (h-3)^2 + (k-3)^2 = h^2 + 4 + 4h$$

$$\Rightarrow k^2 - 10h - 6k + 14 = 0 \text{ or } y^2 - 10x - 6y + 14 = 0.$$

69. (d) Equation of line  $PQ$  (i.e., common chord) is  $5ax + (c-d)y + a + 1 = 0$  .....(i)

Also given equation of line  $PQ$  is

$$5x + by - a = 0 \quad \text{.....(ii)}$$

$$\text{Therefore } \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}; \text{ As } \frac{a+1}{-a} = a$$

$$\Rightarrow a^2 + a + 1 = 0$$

Therefore no real value of  $a$  exists, (as  $D < 0$ ).

70. (c) Suppose the axes are rotated in the anticlockwise direction through an angle  $45^\circ$ . To find the equation of  $L$  w.r.t the new axis, we replace  $x$  by  $x \cos \alpha - y \sin \alpha$  and by  $x \sin \alpha + y \cos \alpha$ , so that equation of line w.r.t. new axes is

$$\Rightarrow 1/1(x \cos 45^\circ - y \sin 45^\circ) + \frac{1}{2}(x \sin 45^\circ + y \cos 45^\circ) = 1$$

Since,  $p, q$  are the intercept made by the line on the coordinate axes. we have on putting  $(p, 0)$  and then  $(0, q)$

$$\Rightarrow \frac{1}{p} = \frac{1}{a} \cos \alpha + \frac{1}{b} \sin \alpha \Rightarrow \frac{1}{q} = -\frac{1}{a} \sin \alpha + \frac{1}{b} \cos \alpha$$

$$\Rightarrow \frac{1}{p} = \frac{1}{1} \cos 45^\circ + \frac{1}{2} \sin 45^\circ$$

$$\Rightarrow \frac{1}{p} = \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$$

$$\therefore p = \frac{2\sqrt{2}}{3}; \quad \therefore \frac{1}{q} = -\frac{1}{1} \sin 45^\circ + \frac{1}{2} \cos 45^\circ$$

$$\frac{1}{q} = \frac{-1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} = -\frac{1}{2\sqrt{2}}, \quad \therefore q = 2\sqrt{2}$$

So intercept made by is assume on the new axis  $(\frac{2\sqrt{2}}{3}, 2\sqrt{2})$ . If the rotation is assume in clockwise direction, so intercept made by the line on the new axes would be  $(2\sqrt{2}, 2\sqrt{2}/3)$ .

71. Ans. (4)

$$\lim_{x \rightarrow \frac{1}{2}} \frac{ax^2 + bx + c}{(2x-1)^2} = \frac{1}{2}$$

$$\Rightarrow ax^2 + bx + c = \frac{1}{2}(2x-1)^2$$

$$\Rightarrow ax^2 + bx + c = 2x^2 - 2x + \frac{1}{2}$$

$$\therefore a = 2, b = -2, c = \frac{1}{2}$$

$$\therefore \lim_{x \rightarrow 2} \frac{(x-2)(x+2)\left(x - \frac{1}{2}\right)}{x-2} = 4 \times \frac{3}{2} = 6$$

72. Its centre is of type  $(c, c)$  and radius is

$$\left| \frac{4c + 3c - 12}{5} \right| = \sqrt{c^2} \Rightarrow c = 6.$$

73. Hyperbola is  $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$   
 $a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, e_1 = \sqrt{1 + \frac{81}{144}} = \sqrt{\frac{225}{144}} = \frac{15}{12} = \frac{5}{4}$   
 Therefore, foci =  $(ae_1, 0) = \left(\frac{12}{5} \cdot \frac{5}{4}, 0\right) = (3, 0)$   
 Therefore, focus of ellipse =  $(4e, 0)$  i.e.  $(3, 0)$   
 $\Rightarrow e = \frac{3}{4}$ . Hence  $b^2 = 16\left(1 - \frac{9}{16}\right) = 7$ .

74. The hyperbola is  $\frac{x^2}{16} - \frac{y^2}{9} = 1$ . We have difference of focal distance =  $2a = 8$ .

75. Any normal is  $y + tx = 6t + 3t^3$ . It is identical with  $x + y = k$  if  $\frac{t}{1} = \frac{1}{1} = \frac{6t + 3t^3}{k}$   
 $\therefore t = 1$  and  $1 = \frac{6 + 3}{k} \Rightarrow k = 9$ .